Hand-In Assignment 6

1. Let $\{x_n\}$ be a sequence of real numbers defined by

   $$x_n = \begin{cases} \dfrac{1}{2k} & if \quad n = 2k-1 \\ \dfrac{1}{2k-1} & if \quad n = 2k \end{cases}$$ where $k \geq 1$ and therefore $n \geq 1$. Set

   $\varepsilon_n = \left(\dfrac{1}{2}\right)^n$ and define $f : R \to R$ by

   $$f(x) = \sum_{n \,:\, x_n < x} \varepsilon_n$$

   (a) Compute $f(0)$, $f(-1)$, $f(1)$, $f(\sqrt{2})$, and $f(1/2)$. [4 pts]

   (b) Determine the set of discontinuities, D(f) , for the function. Justify your claim. [6 pts]

2. Let $f : [a,\ b] \to R$ be increasing, and let $\{x_n\}$ be an enumeration of the discontinuities of f. For each n, let $a_n = f(x_n) - f(x_n-)$ and $b_n = f(x_n+) - f(x_n)$ be the left and right "jumps" in the graph of f, where $a_n = 0$ if $x_n = a$ and $b_n = 0$ if $x_n = b$. Show that $\sum_{n=1}^{\infty} a_n \leq f(b) - f(a)$ and $\sum_{n=1}^{\infty} b_n \leq f(b) - f(a)$. [10 pts]
3. If C is a collection of connected subsets of M, all having a point in common, prove that $\cup\ C$ is connected. [10 pts]
4. Can there be a continuous **onto** function $f : R \to R \setminus Q$? Explain. [10 pts]